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Stanford Research Institute
Menlo Park, California 94025

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Packet Radio Temporary Note #30

Oliver S. Yu and J. W. Sinko
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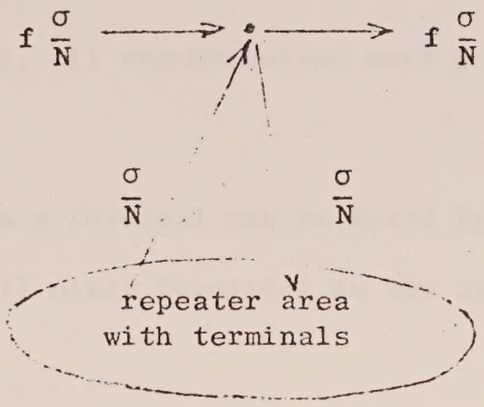
INFORMATION CAPACITY OF MULTI-REPEATER PACKET RADIO SYSTEMS

ABSTRACT: The information capacity of a slotted system with many terminals and a finite number of repeaters is computed. Addressing and confirmation are assumed. Blocking of messages occurs between adjacent repeaters and between a repeater and all terminals within a repeater's area. Blocking does not occur between terminals in different repeater areas. The geometrical arrangements include N repeaters in a circle, N repeaters along a line, and seven repeaters in a hexagonal arrangement. Messages which go outside the local repeater area are equally distributed to the other repeater areas.

The most interesting result is, that for a fixed number of terminals, if the number of messages going outside local areas is not too great, the capacity of the system can be significantly increased by adding more repeaters.

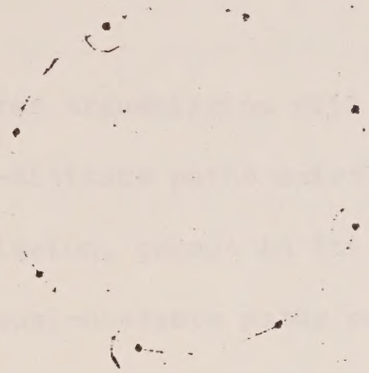
I SYSTEM DESCRIPTION

The systems have N repeaters. Each repeater has an associated set of terminals, and the area containing these terminals is referred to as a repeater area. Each of the links (terminal to repeater, repeater to terminal, and repeater to repeater) have equal capacity. The total rate of information transfer in the system, σ, is measured as the fraction of a single link's unidirectional capacity generated by all the terminals in the system. When there are many repeaters σ may exceed unity because there are parallel links. Each repeater area generates information at a rate of σ/N. A fraction f of σ/N is transmitted to terminals outside the repeater area.



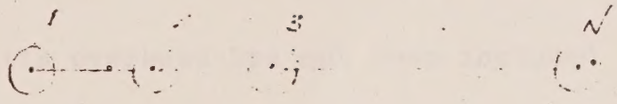
Circular System:

N repeaters are symmetrically situated on a circle.



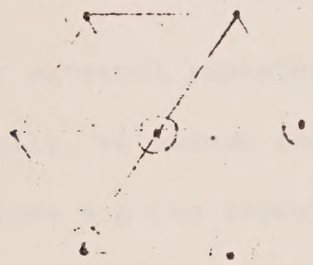
Linear System:

N repeaters along a straight line.



Hexagonal System:

Seven repeaters in a hexagonal arrangement. Repeaters on the perimeter are referred to as vertex repeaters, and the one in the center is called the center repeater.



Operational Assumptions:

1) For each terminal, all communication must go through its associated repeater.

2) Transmission from a terminal can be heard by its associated repeater and will block all other terminals in the same repeater area but no one else.

3) Transmission from a repeater can only be heard by its associated terminals and adjacent repeaters.

4) An addressing mechanism is implied. Inter-area transmission will always take the shortest path. If two equal-distance paths exist, one will be chosen at random for each transmission, except in the case of the hexagonal system, where if two equal-distance paths exist the one along the vertices will be chosen.

5) A confirmation system exists, but its overhead has not been included in the computations.

6) The system is slotted.

7) The patterns of information distribution for external repeaters are identical for all repeater areas. Specifically, we assume that an equal amount of information is to be distributed from a given repeater to every other repeater.

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II DERIVATION OF EQUATIONS FOR INFORMATION CAPACITY

A. Circular System

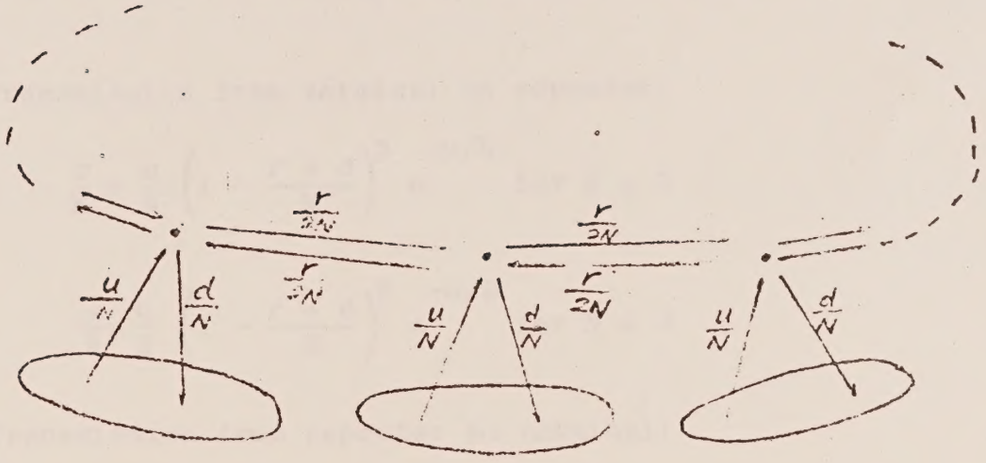
Notations:

d/N is the total traffic rate of probability of message transmission in a time slot from a repeater to its associated terminals.

u/N is the total traffic rate from all the terminals in an area to its associated repeater.

r/(2N) is the traffic rate or probability of message transmission in a time slot from a repeater to one of its adjacent repeaters.

The following diagram depicts the traffic flow of the system.



The traffic for terminals in a given area follows the Poisson distribution with parameter u/N. Therefore, the probability that no packet is transmitted in a given slot is

$$\exp(-u/N).$$

The probability that exactly one packet is transmitted in a given slot is

$$\frac{u}{N} e^{-u/N}$$

The traffic for repeaters follows the binary distribution. Therefore, the probability that no packet is transmitted by a repeater is

$$1 - (r+d)/N.$$

The computations are based on the fact that information transmission is equivalent to successful message traffic transmission. A message traffic transmission is successful if and only if no interfering transmission occurs in the same time slot. A transmission is interfering if it either is originated or can be heard by the intended receiver.

1) Transmission from terminal to repeater:

$$\frac{\sigma}{N} = \frac{u}{N}\left(1 - \frac{r+d}{N}\right)^3 e^{-u/N} \quad \text{for } N \geq 3 \tag{1}$$

$$\frac{\sigma}{2} = \frac{u}{2}\left(1 - \frac{r+d}{2}\right)^2 e^{-u/2} \quad \text{for } N = 2 \tag{1'}$$

2) Transmission from repeater to terminal:

$$\frac{\sigma}{N} = \frac{d}{N} e^{-u/N} \tag{2}$$

3) Transmission from a repeater to one of its adjacent repeaters:

The successful message transmission rate is

$$\frac{r}{2N}\left(1-\frac{r+d}{N}\right)^2 e^{-u/N} \quad \text{for } N \geq 3 \tag{3}$$

$$\frac{r}{2N}\left(1-\frac{r+d}{N}\right) e^{-u/N} \quad \text{for } N = 2 \tag{3'}$$

Since each repeater will on the average transmit at an information rate of $[N/2]\, f\sigma/(N(N-1))$ generated from its own area to one of its two adjacent repeaters with equal fractions of the information to be successively transmitted to each of the $[N/2]$ repeaters along that direction, it is clear that the total information rate to be transmitted (including information from other repeaters in the opposite directions) to that particular adjacent repeater is the following sum:

N odd, $\quad (1+2+3+\ldots+[N/2])f\sigma \ \frac{1}{N(N-1)}$

N even, $\quad (\frac{1}{2}+1\frac{1}{2}+2\frac{1}{2}+\ldots+N/2-\frac{1}{2})f\sigma \ \frac{1}{N(N-1)}$

where $[N/2]$ denotes the integer part of $N/2$. Through simplification, we obtain the following unified formula for the information transmission rate:

$$\frac{\left(\left[\frac{N}{2}\right]\left[\frac{N}{2}+1\right]+\left[\frac{N-1}{2}\right]\left[\frac{N+1}{2}\right]\right) f\sigma}{4N(N-1)} \tag{4}$$

By equating (3) and (4) and combining with (1) and (2), we have three nonlinear equations for the three unknowns: d, r, and u. Again through simplification, we have

$$u = N \ln (d/\sigma)$$

$$r = f'(du^2)^{1/3} \quad \text{for } N \geq 3, \quad r = f(du)^{1/2} \quad \text{for } N = 2$$

$$(1-\frac{d}{N})u^{1/3} = (1+\frac{f'u}{N})d^{1/3} \quad \text{for } N \geq 3, \quad (1-\frac{d}{2})u^{1/2} = (1+\frac{fu}{2})d^{1/2} \quad \text{for } N = 2$$

where $f' = \dfrac{\left[\frac{N}{2}\right]\left[\frac{N}{2}+1\right]+\left[\frac{N-1}{2}\right]\left[\frac{N+1}{2}\right]}{2(n-1)} f$ and $[\ ]$ denotes integer part.

Further Unification

Suppose that we let

$$S = \sigma/N, \quad U = u/N, \quad R = r/N, \quad D = d/N. \qquad N \geq 3$$

Then the three nonlinear equations may be rewritten as:

$$S = U(1-R-D)^3 e^{-U}$$

$$S = D e^{-U}$$

$$f'S = R(1-R-D)^2 e^{-U}$$

Therefore, the solution of this set of nonlinear equations is applicable to any value of N with appropriate interpretations for f', S, U, R, and D.

B. Linear System

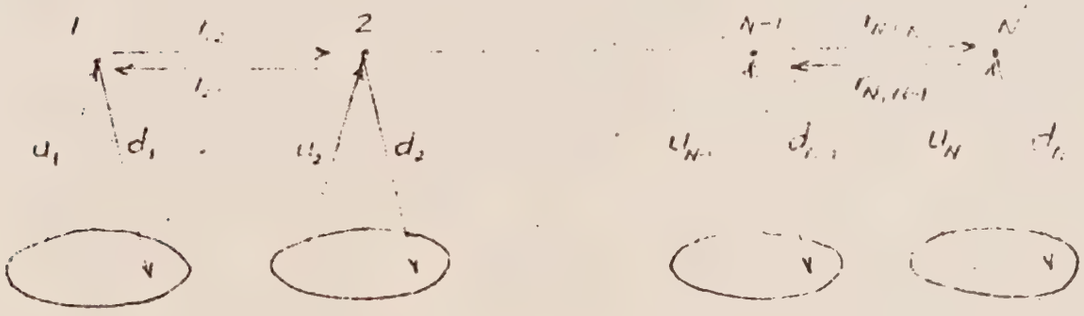
Notations:

d_i is the total traffic rate or probability of message transmission in a time slot from repeater i to its associated terminals.

r_{ij} is the traffic rate or probability of message transmission in a time slot from repeater i to repeater j, with j being either i-1 or i+1.

u_i is the total traffic rate from all the terminals in area i.

The following diagram depicts the traffic flow of the system.



Computational Procedure:

For repeater area i ($i = 1,2,\ldots, \left[\frac{(N+1)}{2}\right]$)*, let

$$r_i = r_{i,i-1} + r_{i,i+1} \text{ and } r_{j1} = r_{1j} = r_{jN} = r_{Nj} = d_j = u_j = 0$$

for $j \leq 0$ or $j \geq N + 1$

1) Transmission from terminal to repeater

$$\frac{\sigma}{N} = u_i e^{-u_i} (1-r_{i-1} - d_{i-1}) (1-r_i-d_i) (1-r_{i+1} - d_{i+1}) \qquad (5)$$

2) Transmission from repeater to terminal

$$\frac{\sigma}{N} = d_i e^{-u_i} \qquad (6)$$

3) Transmission from repeater i to repeater i-1

$$(i-1)(N-i+1) f \frac{\sigma}{N(N-1)} = r_{i,i-1} e^{-u_{i-1}} (1-r_{i-1} -d_{i-1})(1-r_{i-2} -d_{i-2}) \qquad (7)$$

4) Transmission from repeater i to repeater i+1

$$i(N-i) f \frac{\sigma}{N(N-1)} = r_{i,i+1} e^{-u_{i+1}} (1-r_{i+1} -d_{i+1})(1-r_{i+2} -d_{i+2}) \qquad (8)$$

The factor (i-1) (N-i+1) in Equation (6) arises from the fact that each of the N-i+1 repeaters with numbers higher than i-1, i.e., repeaters i, i+1, i+2,...,N transmits at an information rate of $(i-1)f\sigma/(N(N-1))$ through repeater i to be distributed equally among the i-1 repeaters with numbers lower than i, i.e., repeaters 1, 2, ..., i-1. The factor i (N-i) in Equation (7) arises from the fact that each of the i repeaters with numbers lower than i+1 transmits at an information rate of $(N-i)\ f\ \sigma/(N(N-1))$ through repeater i to be distributed equally among the N-i repeaters with number higher than i.

For $i > \left[\frac{(N+1)}{2}\right]$, let $j = i - \left[\frac{(N+1)}{2}\right]$, then by symmetry,

$$d_i = d_j$$

*[ ] denotes integer part

$$u_i = u_j$$

$$r_{i,i-1} = r_{j,j+1}$$

$$r_{i,i+1} = r_{j,j-1}$$

and the set of equations for repeater area i is identical to that for repeater j.

C. Hexagonal System

Notations:

d_v is the total message transmission rate or probability of message transmission in a time slot from vertex repeater to its associated terminals.

d_c is the total message transmission rate or probability of message transmission in a time slot from center repeater to its associated terminals.

r_{vv} is the message transmission rate or probability of message transmission in a time slot from a vertex repeater to one of its adjacent vertex repeaters.

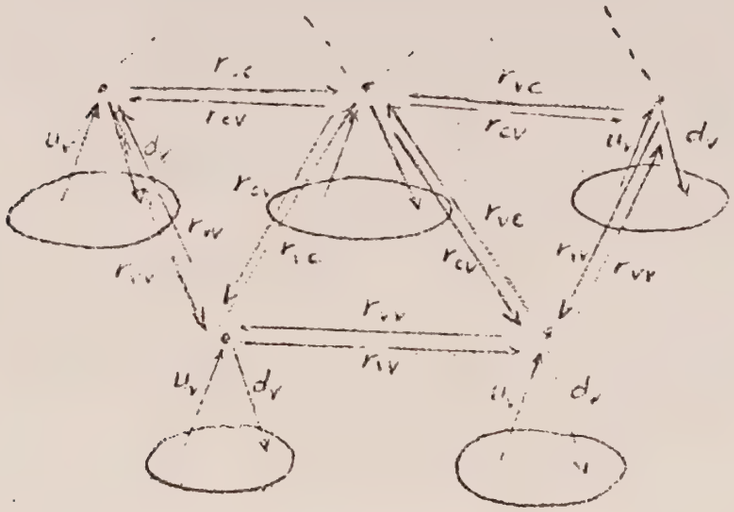
r_{vc} is the message transmission rate or probability of message transmission in a time slot from a vertex repeater to the center repeater.

r_{cv} is the message transmission rate or probability of message transmission in a time slot from the center repeater to one of the vertex repeaters.

u_v is the total message transmission rate from all the terminals in a vertex repeater area to the repeater.

u_c is the total message transmission rate from all the terminals in a vertex repeater area to the center repeater area.

The following diagram depicts the traffic flow of the system.



Let $r_v = 2r_{vv} + r_{vc}$, $r_c = 6r_{cv}$. (9)

1) Vertex Repeater:

a) Transmission from terminal to repeater

$$\frac{\sigma}{7} = u_v\, e^{-u_v} (1-r_v-d_v)^3 (1-r_c-d_c)$$

b) Transmission from repeater to terminal

$$\frac{\sigma}{7} = d_v\, e^{-u_v} \tag{10}$$

c) Transmission from vertex repeater to center repeater

$$\frac{f\sigma}{21} = r_{vc}\, e^{-u_c} (1-r_v-d_v)^5 (1-r_c-d_c) \tag{11}$$

d) Transmission from vertex repeater to one of its adjacent vertex repeate

$$\frac{f\sigma}{14} = r_{vv} e^{-u_v} (1-r_v-d_v)^2 (1-r_c-d_c) \tag{12}$$

2) Center Repeater:

a) Transmission from terminal to repeater

$$\frac{\sigma}{7} = u_c e^{-u_c} (1-r_v-d_v)^6 (1-r_c-d_c) \tag{13}$$

b) Transmission from repeater to terminal

$$\frac{\sigma}{7} = d_c e^{-u_c} \tag{14}$$

c) Transmission from center repeater to vertex repeater

$$\frac{f\sigma}{21} = r_{cv} e^{-u_v} (1-r_v-d_v)^3 \tag{15}$$

The factor on the right-hand sides of Equations (3), (4), and (7) are results of successive message transmissions on directed routes defined by the operational assumptions.

Consequently, we have 7 equations for 7 unknowns: d_v, d_c, u_v, u_c, r_{vv}, r_{vc}, r_{cv}.

III RESULTS AND DISCUSSION

For a given system configuration with a specific number of repeaters and a given value for the fraction of messages going outside each repeater area, we can solve the associated set of nonlinear equations to obtain the traffic rates (up, down, and relayed) as functions of the information transmission rate σ. As an example, these functions for a two-repeater system are shown in Figures 1 through 3 for various values of f. From these curves, the maximum information transmission rate σ_{max} can be easily obtained. As would be expected, for a given system, the maximum information transmission rate, or the system information capacity decreases as the fraction of information leaving each repeater area increases as shown by the example of the 5-repeater circular system in Figure 4.

To compare the information capacities of the three systems, the individual maximum information transmission rates, each as a function of the number of repeaters in the system and the fraction of information leaving each repeater area, for these systems are shown combinedly in Figure 5. When f=0, the straight line of repeaters is slightly better than the circular arrangement. This is because the two end repeaters do not interfere with each other. However, as the fraction of information leaving the repeater area increases, the circular arrangement becomes more advantageous. This is because relayed messages are a major factor in limiting system capacity, and in the circular system the average number of times a message is relayed is lower. Also, the relay traffic load is evenly distributed between repeaters in the circular system while in the straight line system the relay traffic at the central repeaters is much higher than at the repeaters near the ends. The hexagonal system (compared to circular and straight line system with N=7) is less effective when f=0 because each repeater's up and down traffic can be blocked by three or more other repeaters. However, as f increases, the increased number of links in the system enables the average number of relay steps to be reduced, and gives the system a significantly higher capacity than the circular and straight line systems. These rather optimistic results for the hexagonal system indicates that the added links gained in two dimensional arrangements of terminals may yield even larger gains in capacity. This

system information capacity increases with the number of repeaters. However, because the total number of terminals is fixed, this means the amount of information each terminal can transmit increases with the number of repeaters. Furthermore, in a practical system a terminal is more likely to communicate with nearby terminals than with far away terminals. In such systems, f will increase slowly with N. Therefore, the increase in information capacity for such systems (and their terminals) should be greater than that indicated in Figure 8. In fact, the increase should approach those shown in Figure 5 where f is held as a constant as N varies.

In summary, we conclude that for a fixed number of terminals, if the number of messages going outside local areas is not too great, the information capacity of the system can be significantly increases by adding more repeaters, and the system information capacity can also be increased by a two-dimensional arrangement of the repeaters.

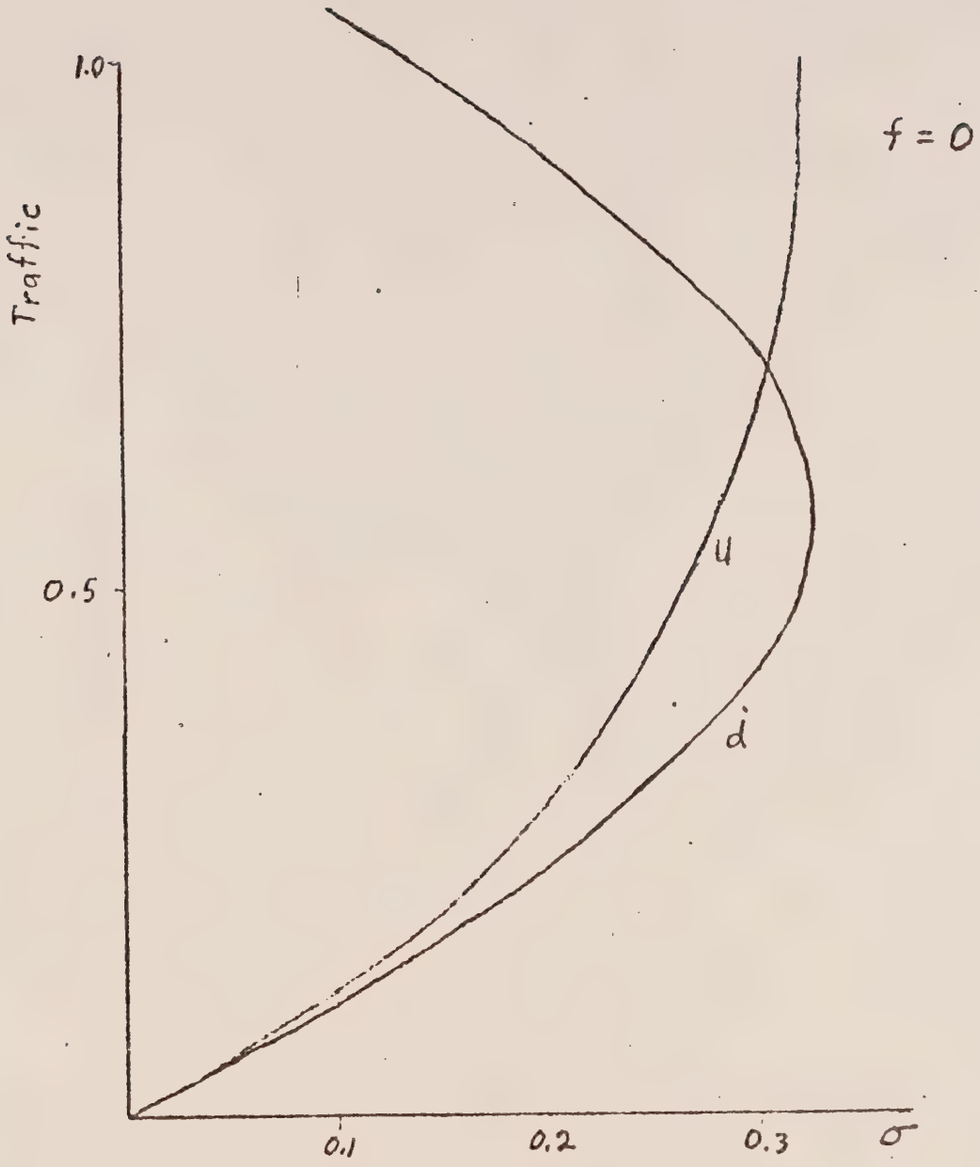


Figure 1

Traffic rates versus information
transmission rate for a system with two repeaters.

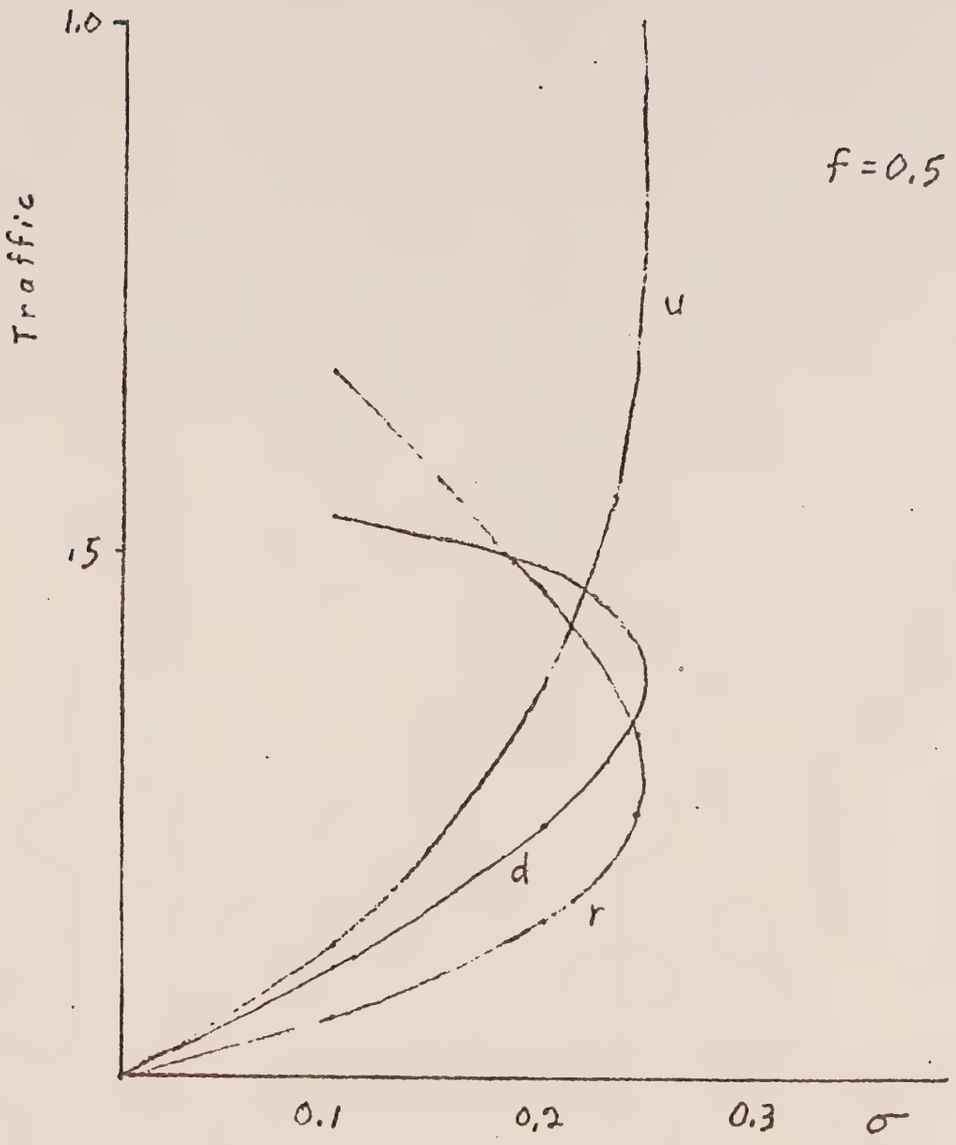


Figure 2

Traffic rates versus information transmission rate for a system with two repeaters.

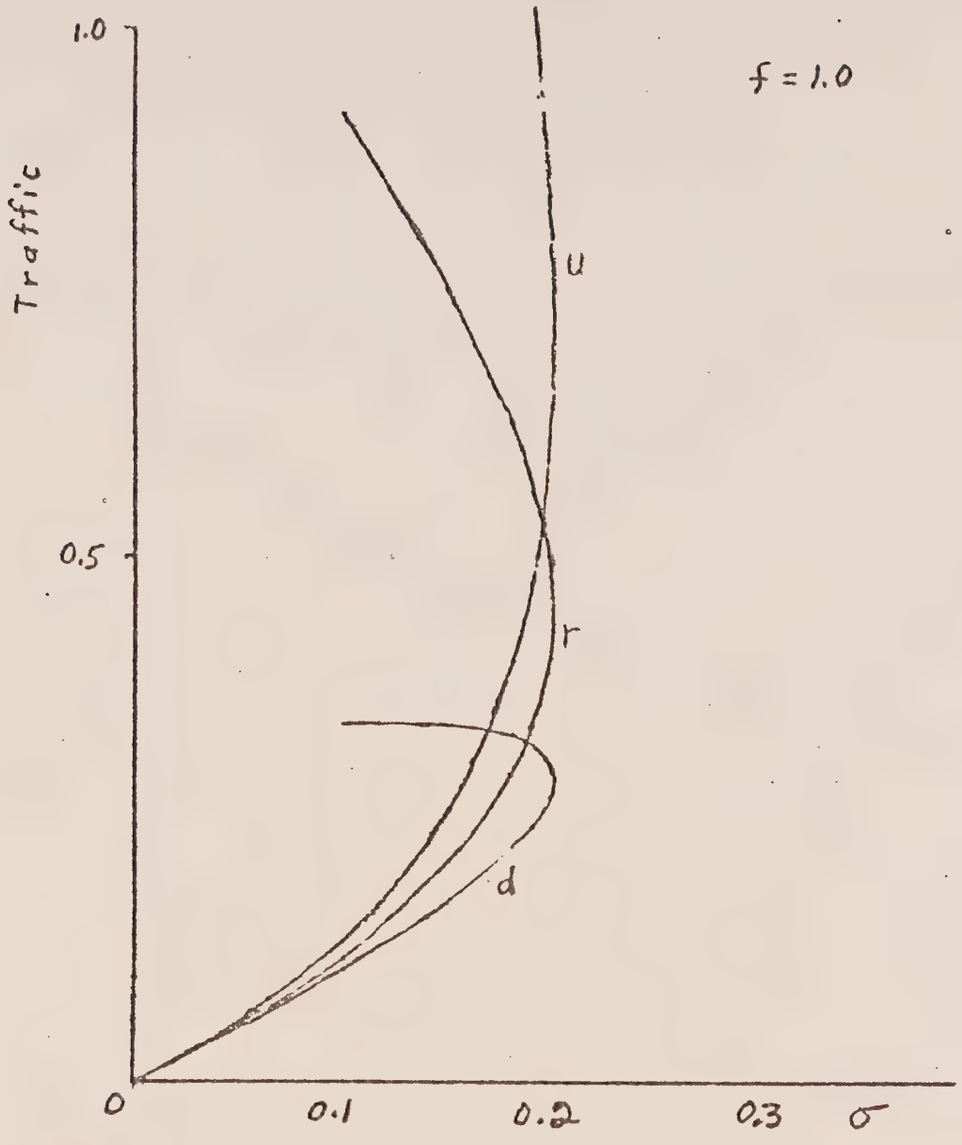


Figure 3

Traffic rates versus information transmission rate for a system with two repeaters.

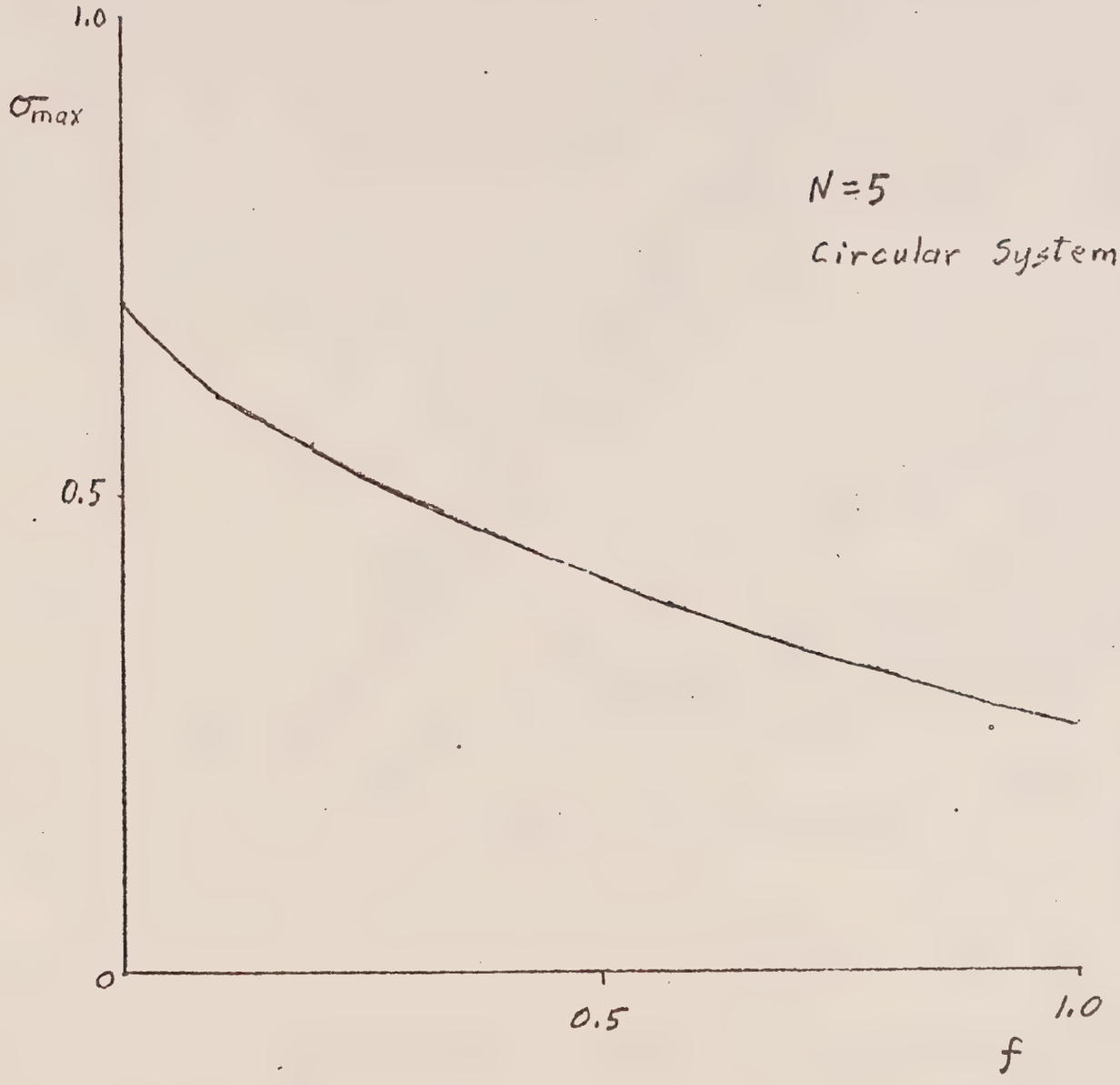


Figure 4

Maximum information transmission rate as a function of fraction of messages going outside the terminal area for a system with five repeaters.

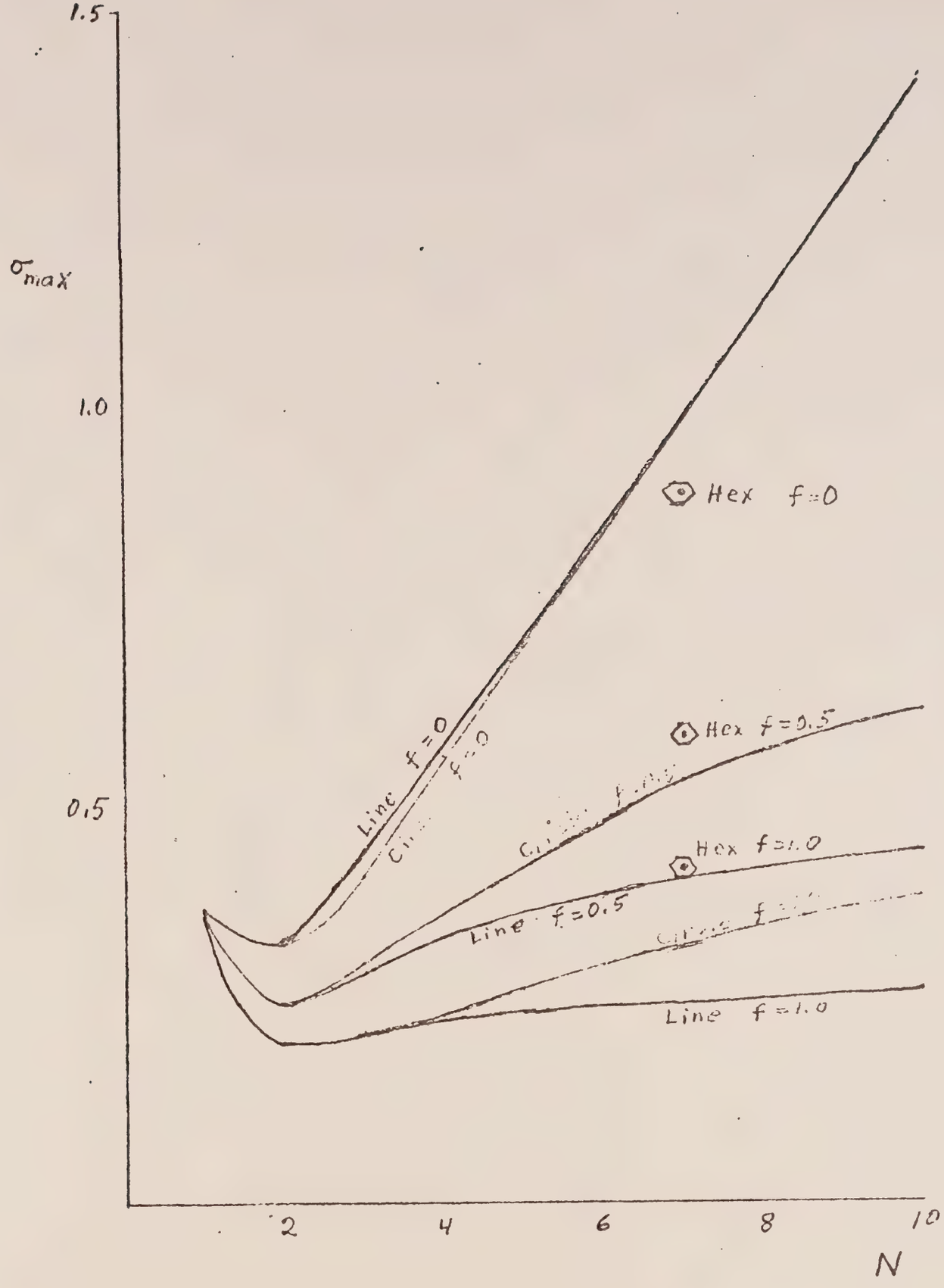


Figure 5

Maximum information transmission rate for different numbers of repeaters and fractions of messages going outside the repeater area.

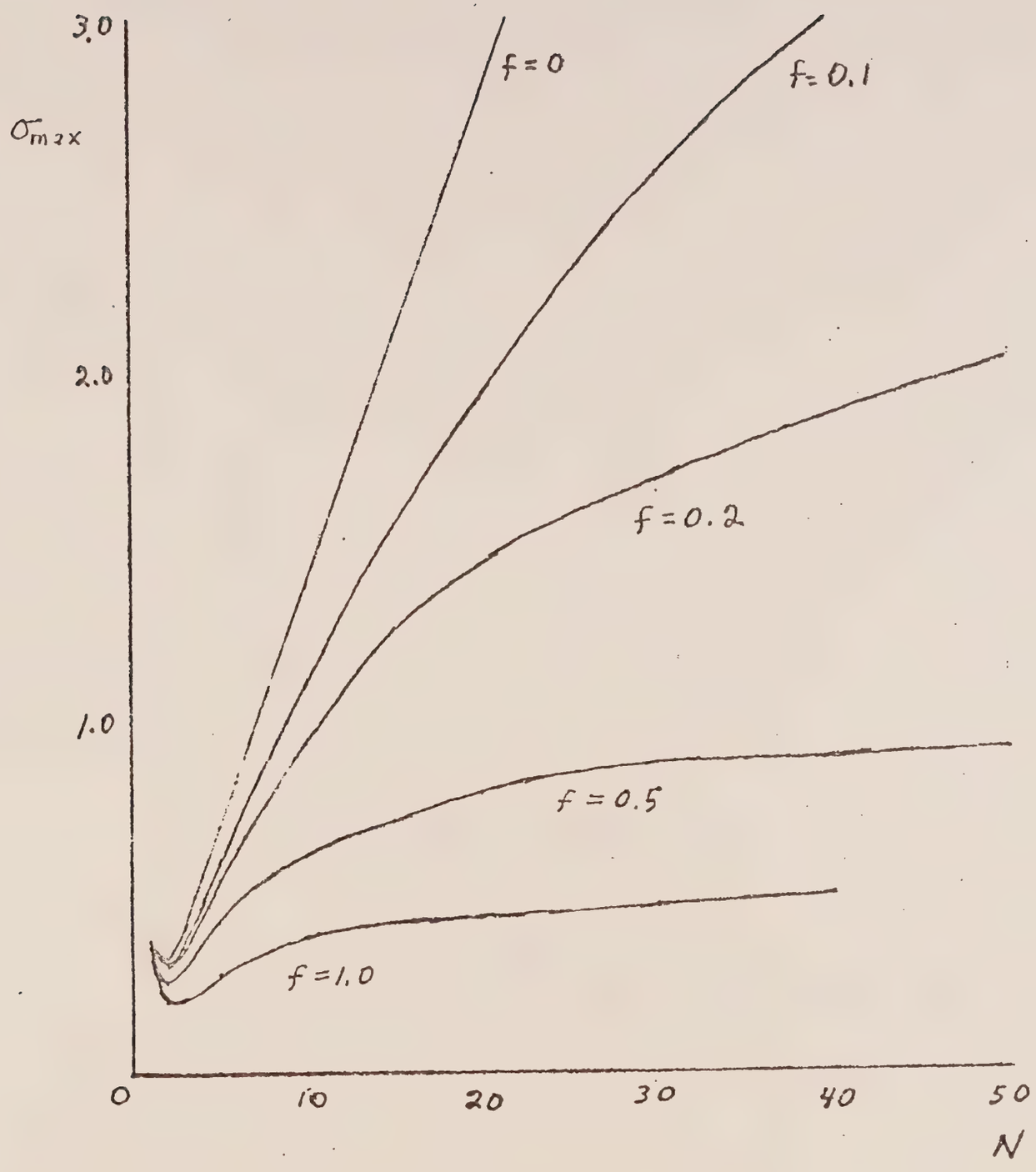


Figure 6

Maximum information transmission rate for a circular system with different numbers of repeaters and fractions of messages going outside the repeater area.

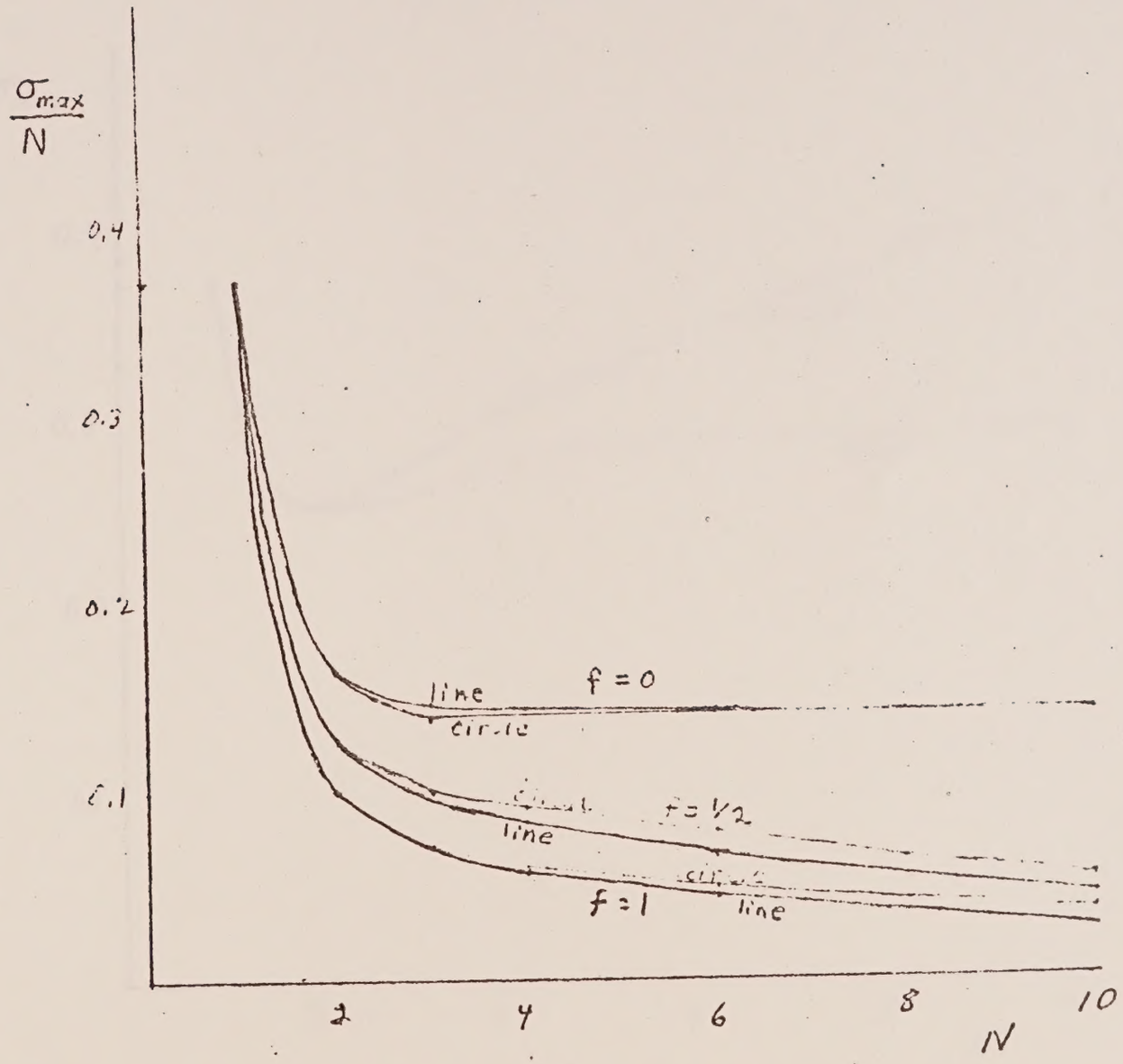


Figure 7

Information capacity for each repeater area
as a function of number of repeaters.
The number of terminals is proportioned
to the number of repeaters.

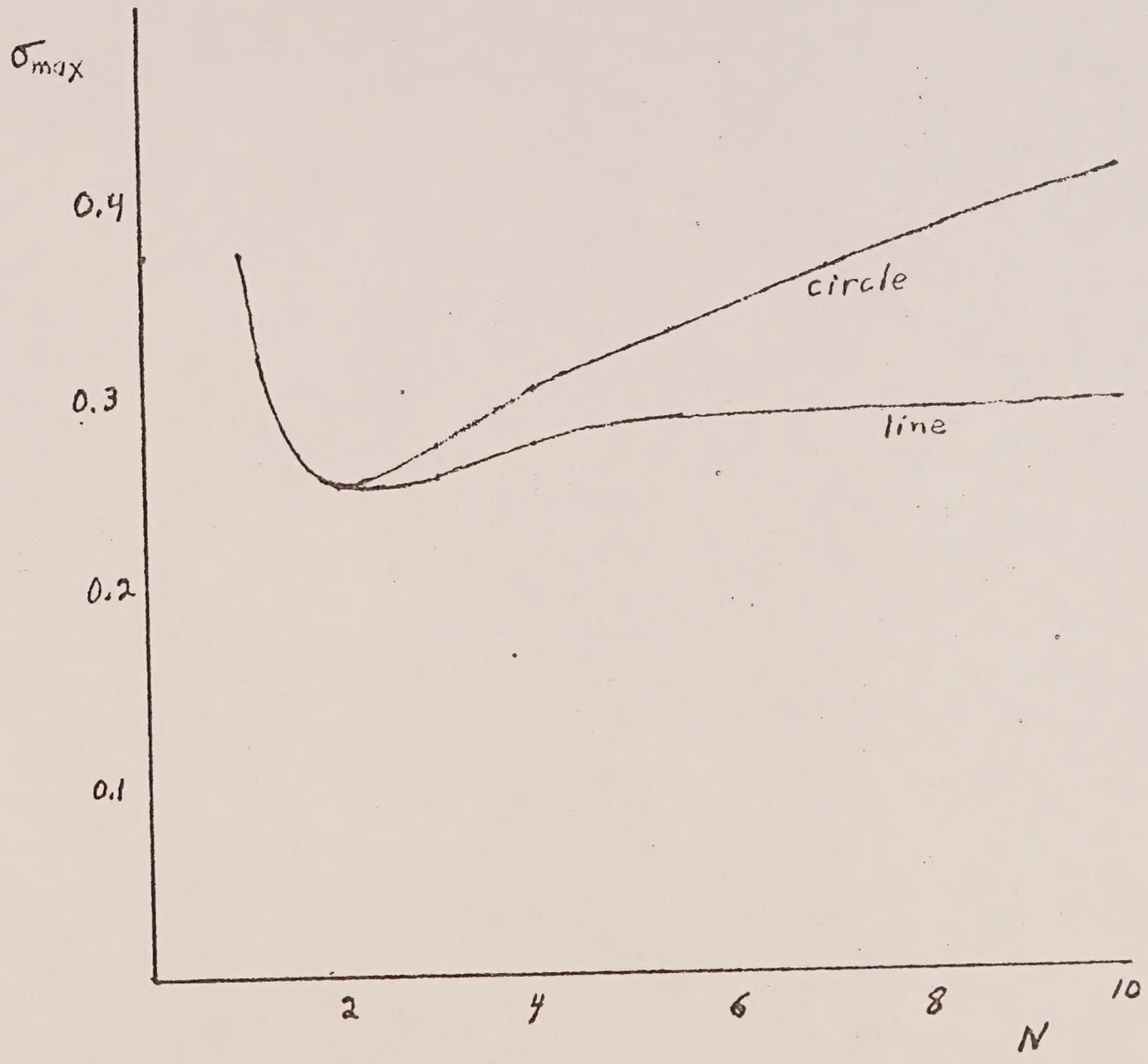


Figure 8

Maximum information transmission rate versus number of repeaters when messages from each terminal are equally distributed to all other terminals.